MATH 2020 HW1 solution

Questions : 15.1 : 10, 20, 27
15.2 : 11, 22, 31, 40, 49, 61, 78

15.1.10). $\int_0^1 \int_1^2 xye^x \, dy \, dx$

$= \int_0^1 \left[x\left(\frac{y^2}{2}\right) e^x \right]_{y=1}^{y=2} dx$

$= \left[\frac{y^2}{2}\right]_{y=1}^{y=2} \int_0^1 xe^x \, dx$

$= \left[\frac{y^2}{2}\right]_{y=1}^{y=2} \left[xe^x - e^x \right]_{x=0}^{x=1}$

$= \left(\frac{3}{2}\right)(1)$

$= \frac{3}{2}$

15.1.20) $\iint_R \frac{y}{xy+1} \, dA$

$= \int_0^1 \int_0^1 \frac{y}{xy+1} \, dx \, dy$

$= \int_0^1 \left[\tan^{-1} xy \right]_{x=0}^{x=1} dy$

$= \int_0^1 \tan^{-1} y \, dy$

$= \left[y \tan^{-1} y - \frac{1}{2} \log(1+y^2) \right]_0^1$

$= \frac{\pi}{4} - \frac{1}{2} \log 2$

15.1.27) required volume

$= \int_0^{\pi/2} \int_0^{\pi/4} 2 \sin x \cos y \, dy \, dx$

$= \int_0^{\pi/2} \left[2 \sin x \sin y \right]_{y=0}^{y=\pi/4} dx$

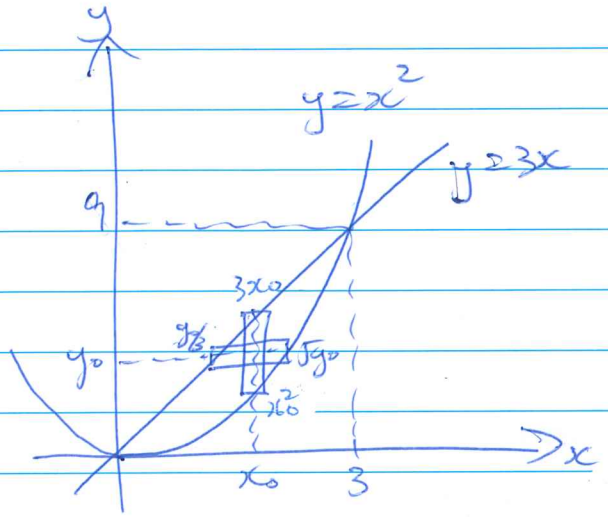
$= 2 \left[\sin y \right]_{y=0}^{y=\pi/4} \int_0^{\pi/2} \sin x \, dx$

$= 2 \left[\sin y \right]_{y=0}^{y=\pi/4} \left[-\cos x \right]_{x=0}^{x=\pi/2}$

$= 2 \left(\frac{\sqrt{2}}{2}\right)(1)$

$= \sqrt{2}$

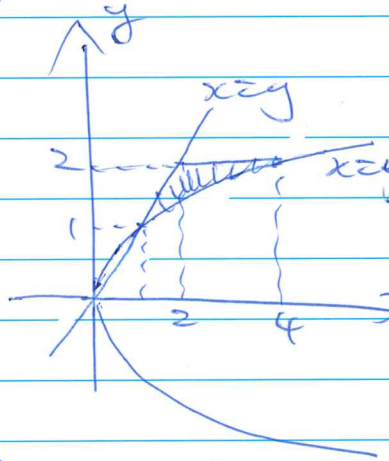
15.2.11)



a) $\int_0^3 \int_{x^2}^{3x} dy\,dx$

b) $\int_0^9 \int_{y/3}^{\sqrt{y}} dx\,dy$

15.2.22)



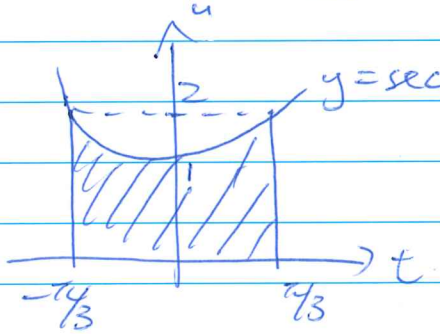
$\int_1^2 \int_y^{y^2} dx\,dy$

$= \int_1^2 (y^2 - y)\,dy$

$= \left[\frac{1}{3}y^3 - \frac{1}{2}y^2\right]_{y=1}^{y=2}$

$= \frac{5}{6}$

15.2.31)



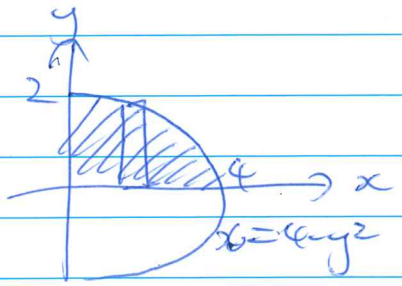
$\int_{-\pi/3}^{\pi/3} \int_0^{\sec t} 3\cos t\,du\,dt$

$= \int_{-\pi/3}^{\pi/3} 3\cos t \sec t\,dt$

$= \int_{-\pi/3}^{\pi/3} 3\,dt$

$= 2\pi$

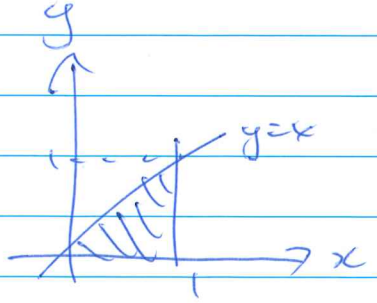
15.2.40)



$\int_0^2 \int_0^{4-y^2} y\,dx\,dy$

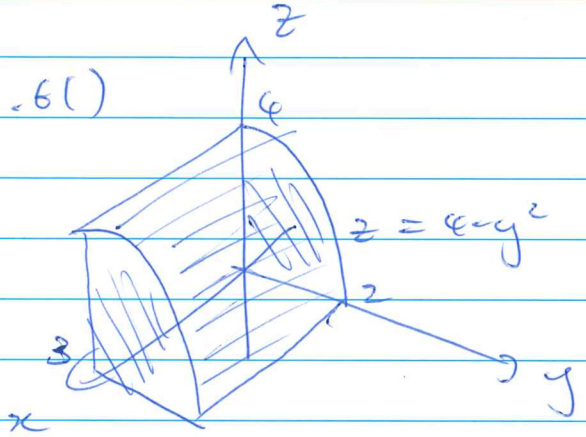
$= \int_0^4 \int_0^{\sqrt{4-x}} y\,dy\,dx$

(5.2.49)



$$\int_0^1 \int_y^1 x^2 e^{xy}\, dx\, dy$$
$$= \int_0^1 \int_0^x x^2 e^{xy}\, dy\, dx$$
$$= \int_0^1 \left[x e^{xy} \right]_{y=0}^{y=x} dx$$
$$= \int_0^1 (x e^{x^2} - x)\, dx$$
$$= \left[\frac{1}{2}(e^{x^2} - x^2) \right]_{x=0}^{x=1}$$
$$= \frac{e}{2} - 1$$

(5.2.61)



required vol
$$= \int_0^3 \int_0^2 (4 - y^2)\, dy\, dx$$
$$= \int_0^3 \left[4y - \frac{1}{3}y^3 \right]_{y=0}^{y=2} dx$$
$$= \int_0^3 \frac{16}{3}\, dx$$
$$= 16.$$

(5.2.78) $\int_0^2 (\tan^{-1} 2x - \tan^{-1} x)\, dx$
$$= \int_0^2 \int_1^2 \frac{x}{1 + x^2 y^2}\, dy\, dx$$
$$= \int_1^2 \int_0^2 \frac{x}{1 + x^2 y^2}\, dx\, dy$$
$$= \int_1^2 \left[\frac{1}{2y^2} \log(1 + x^2 y^2) \right]_{x=0}^{x=2} dy$$
$$= \frac{1}{2} \int_1^2 \frac{1}{y^2} \log(1 + 4y^2)\, dy$$
$$= \frac{1}{2} \left[-\frac{1}{y} \log(1 + 4y^2) + 4 \tan^{-1} 2y \right]_1^2$$
$$= -\frac{1}{4} \log(1 + 4^2) + \frac{1}{2} \log 5 + 2 \tan^{-1} 4 - 2 \tan^{-1} 2$$