Preferential Attachment and Homology

Evolution of Higher Dimensional Interactions

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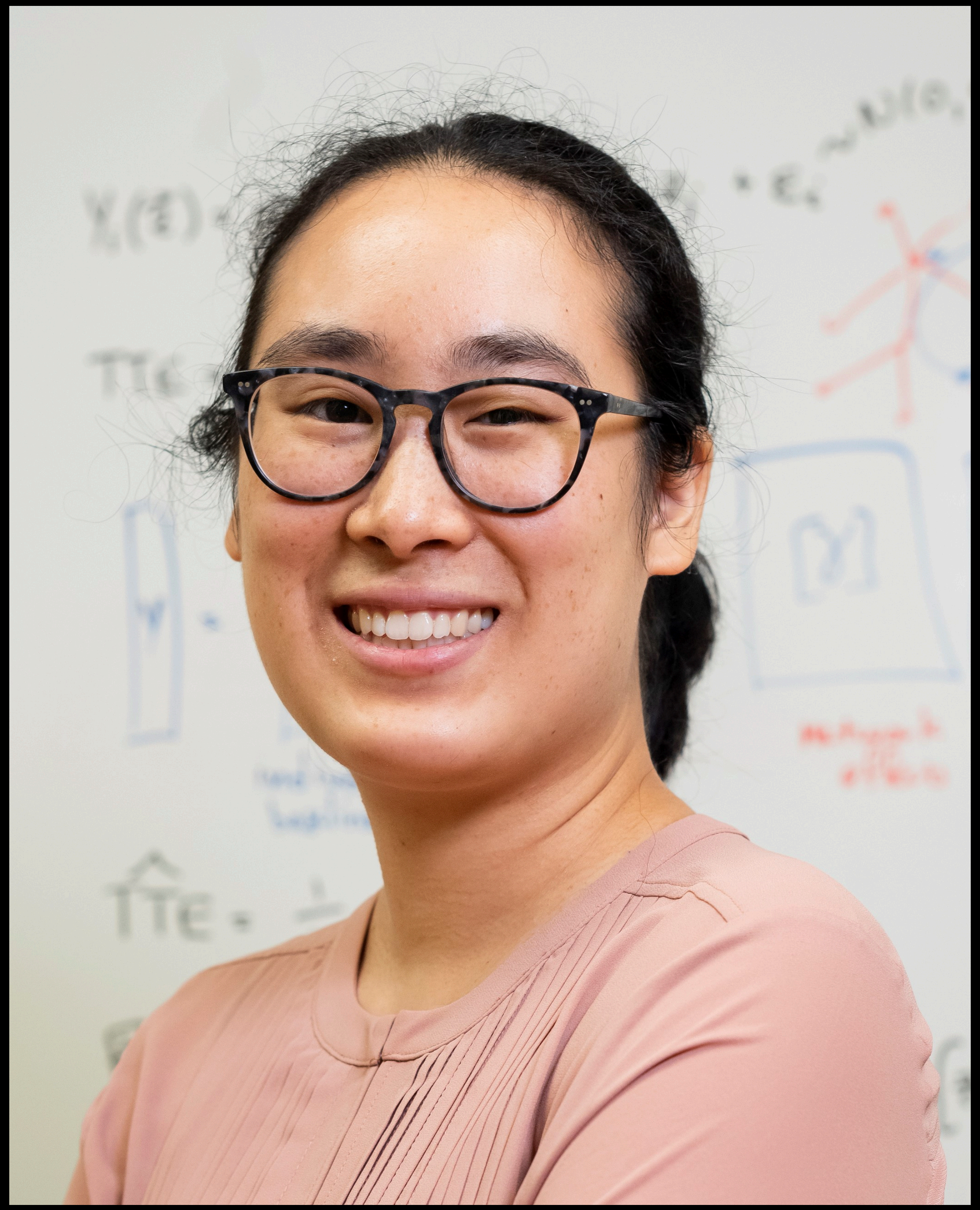
Collaborators



Christina Lee Yu

Gennady Samorodnitsky

Caroline He

Agenda

- What is preferential attachment?
- What is homology?
  - Why people care?
- What we know about the homology of preferential attachment complexes

Preferential Attachment

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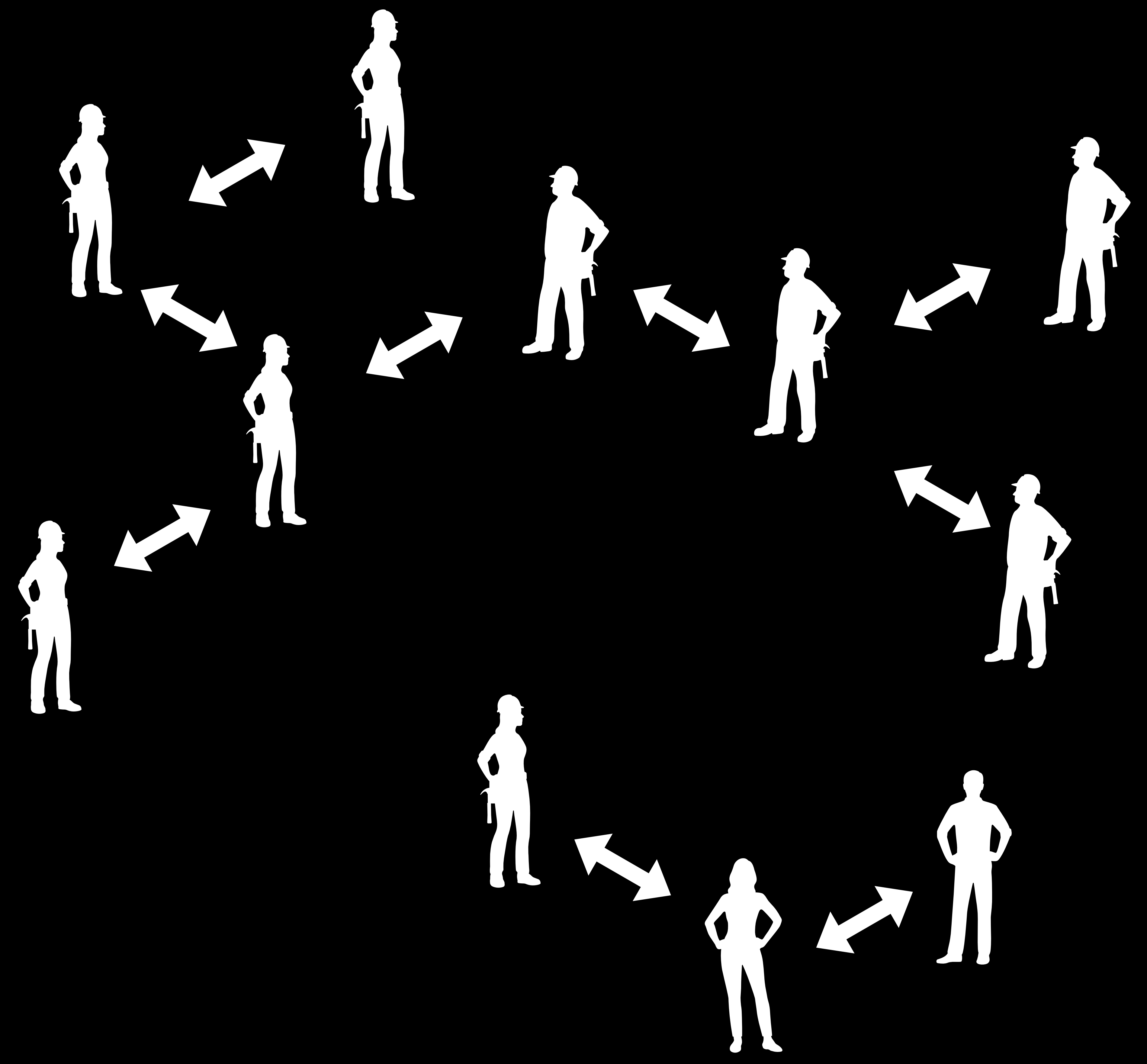
[Albert and Barabasi 1999]



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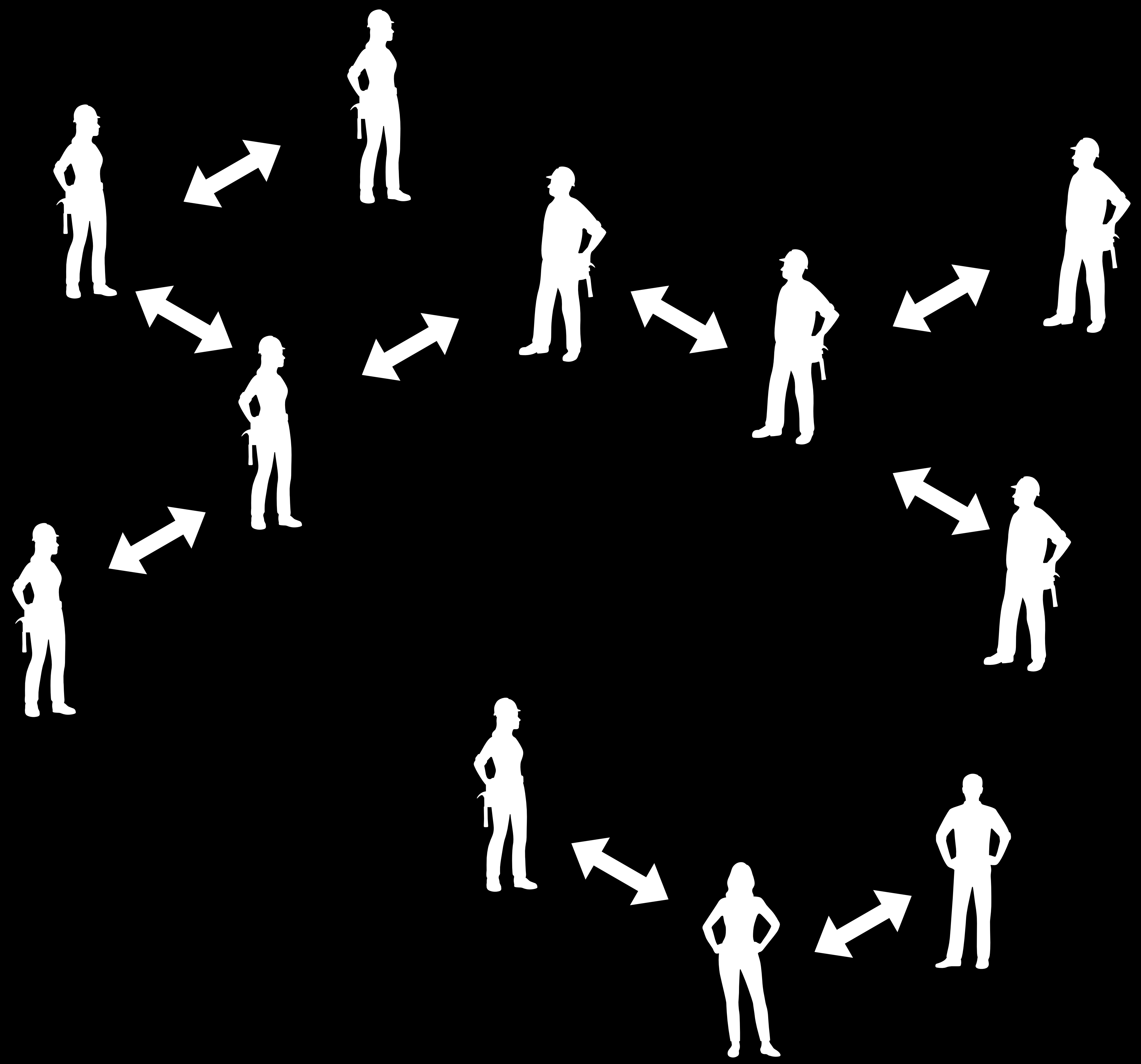
[Albert and Barabasi 1999]



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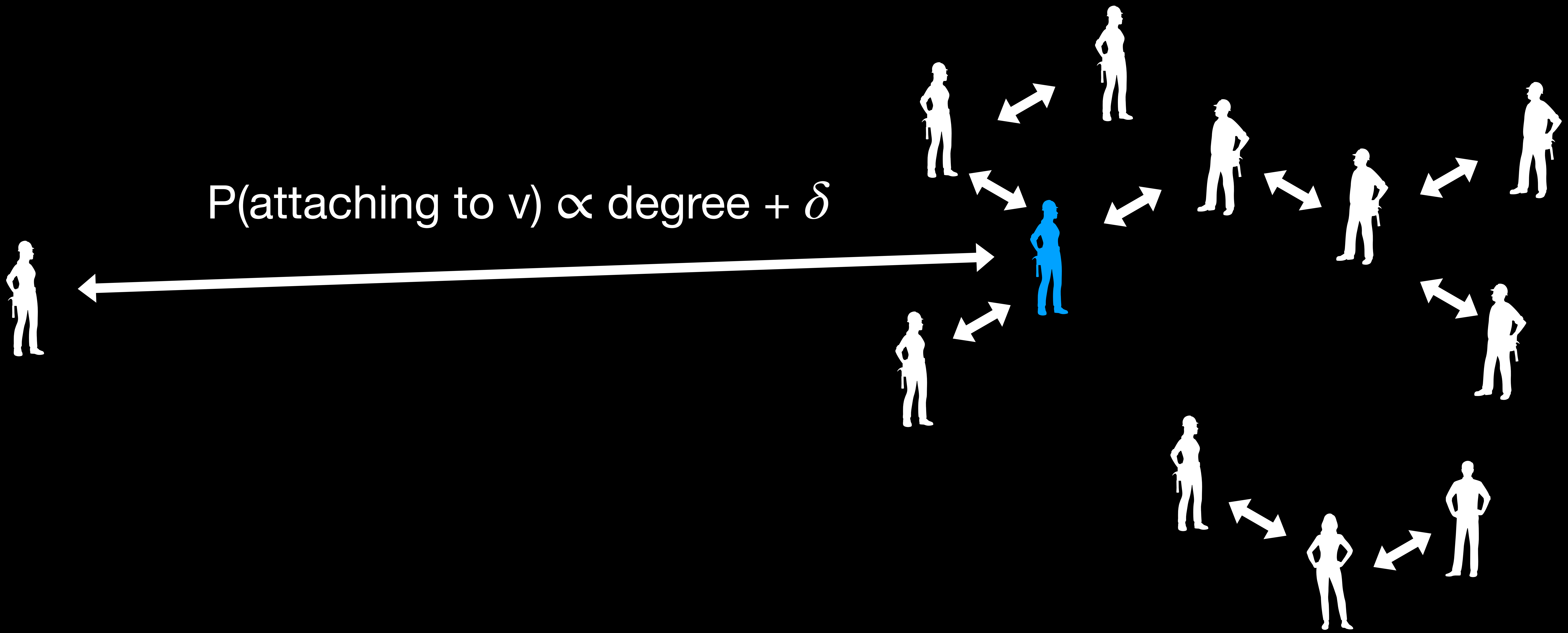
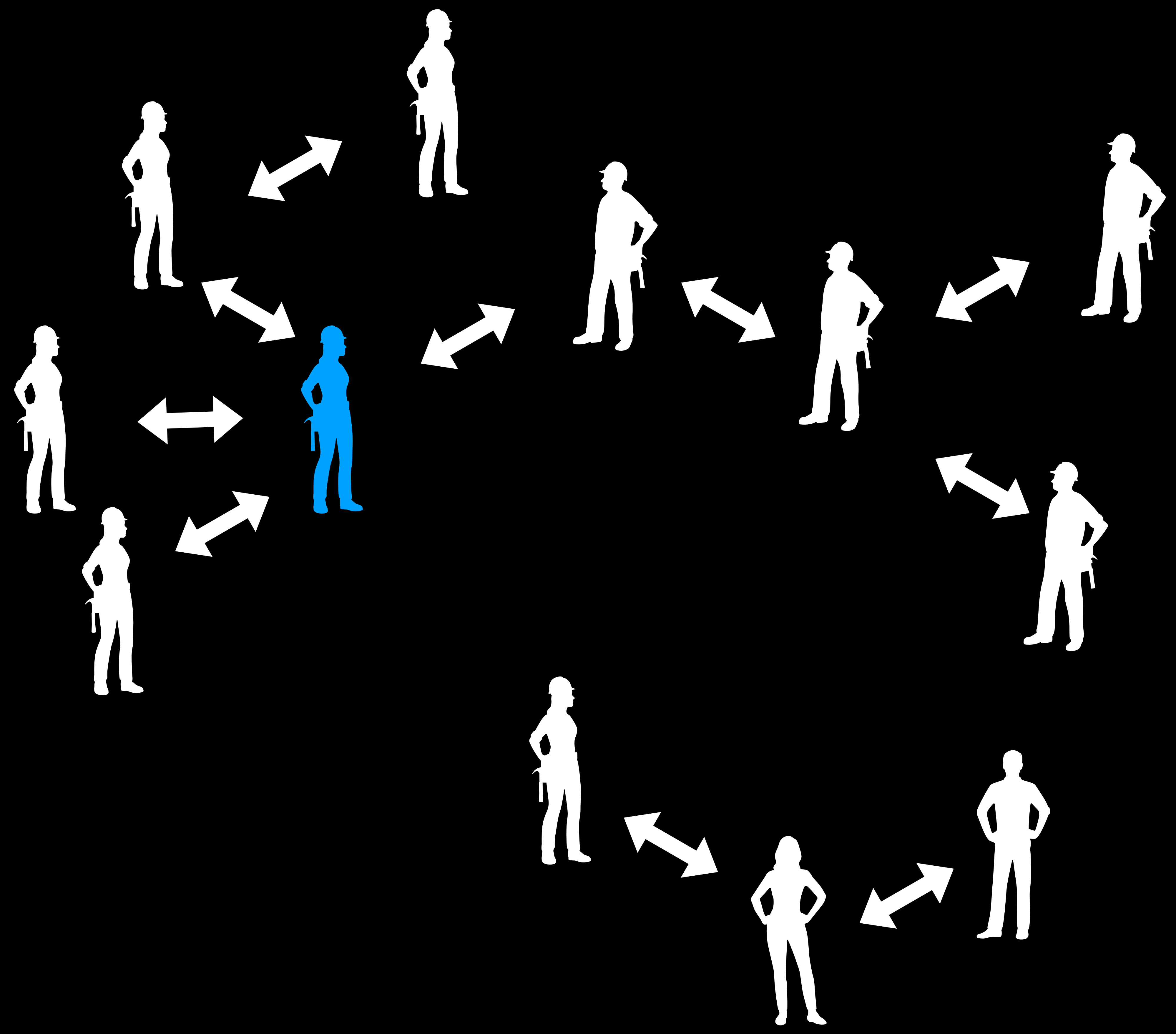


Preferential Attachment

[Albert and Barabasi 1999]

P(attaching to v) $\propto$ degree + δ



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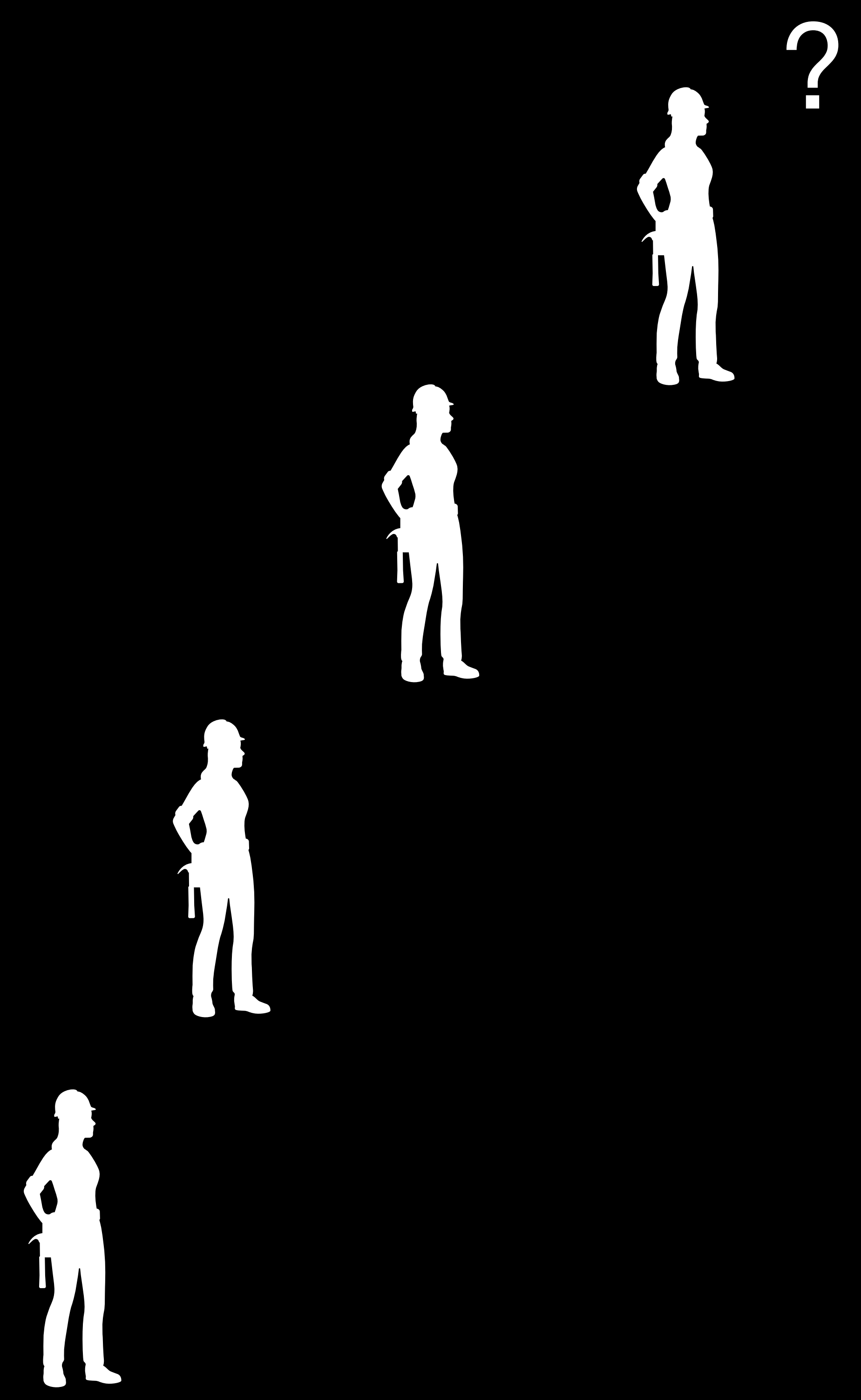

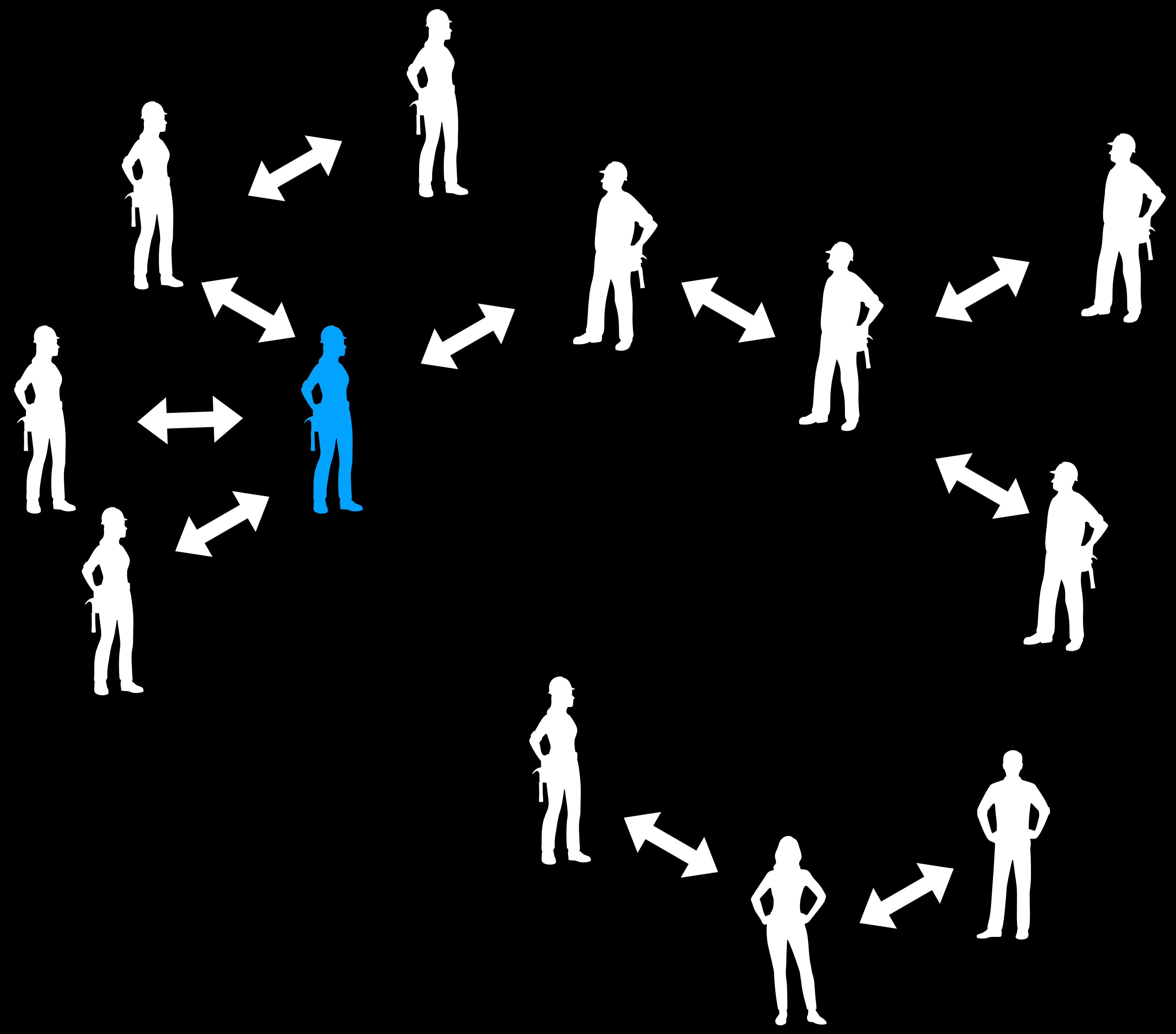

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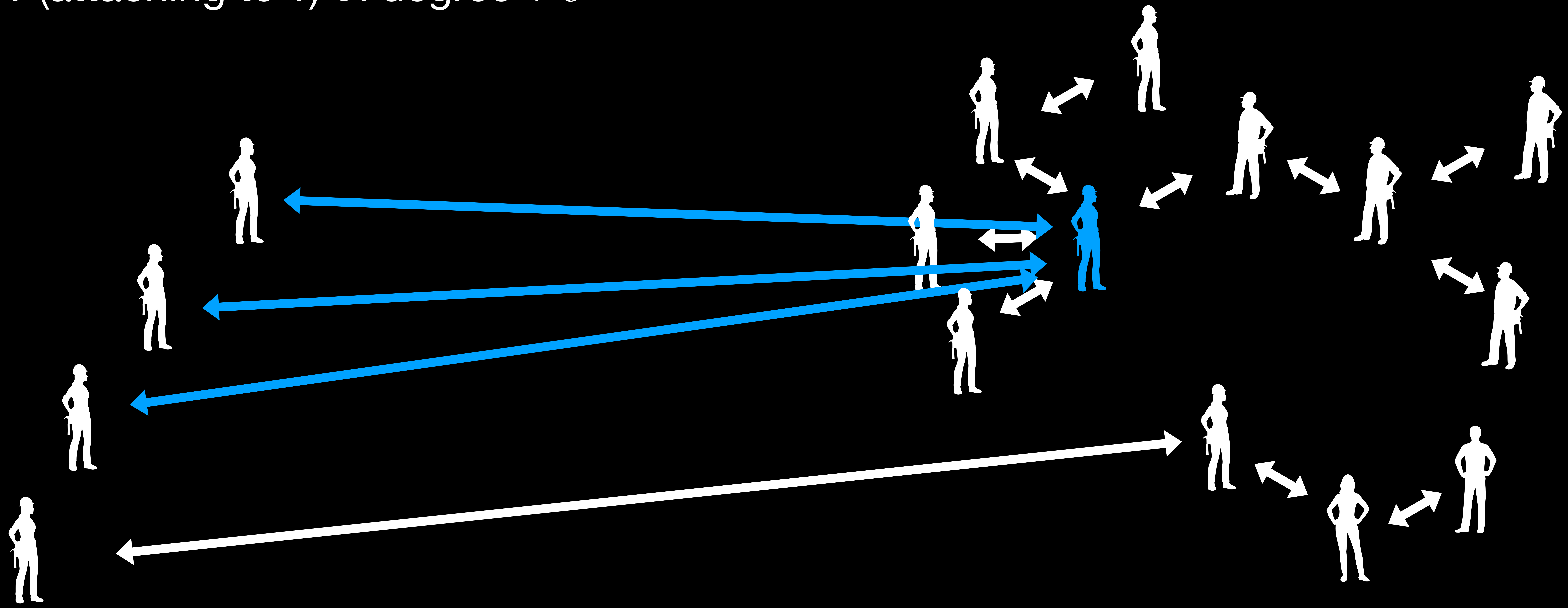

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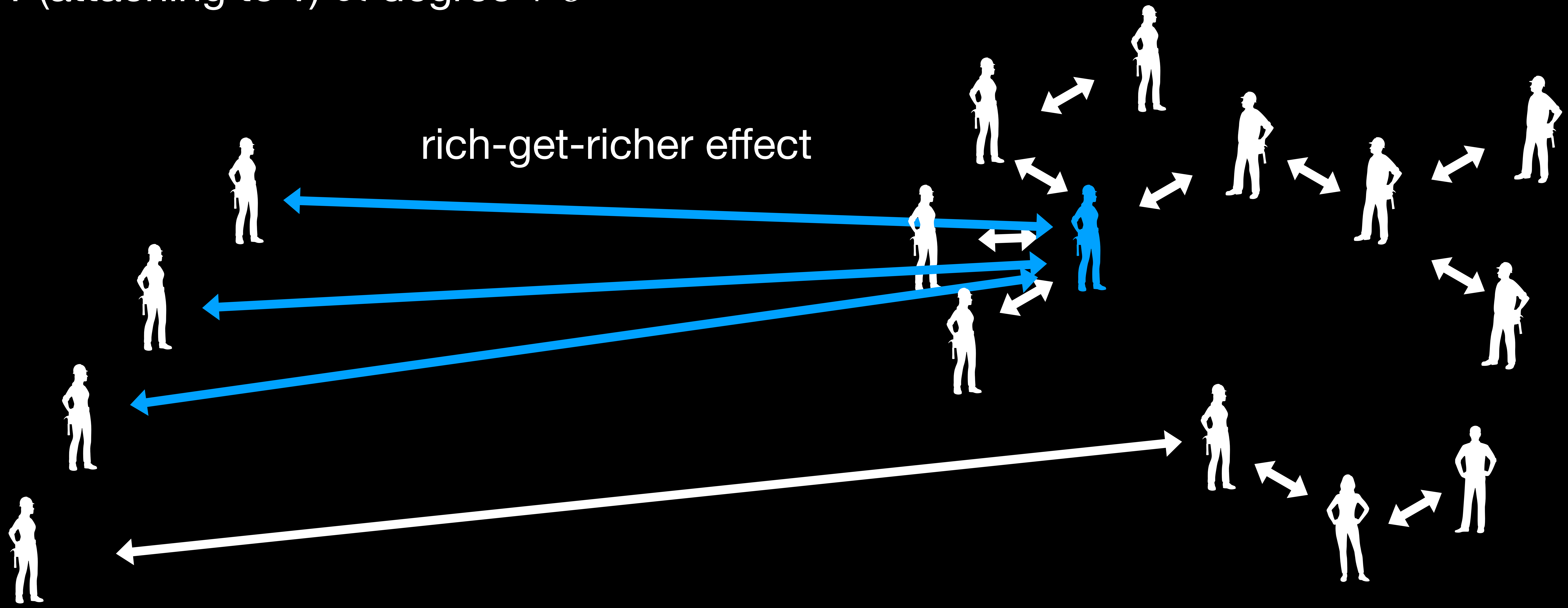

What do we know?

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- heavy tailed degree distribution [Albert and Barabasi 1999]

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- triangle counts and clustering coefficient [Bollobas and Ridden 2002, Prokhorenkova et al 2013]
- subgraph counts [Garavaglia and Steghuis 2019]
- and more…

Triangles, Tetrahedra and Topology

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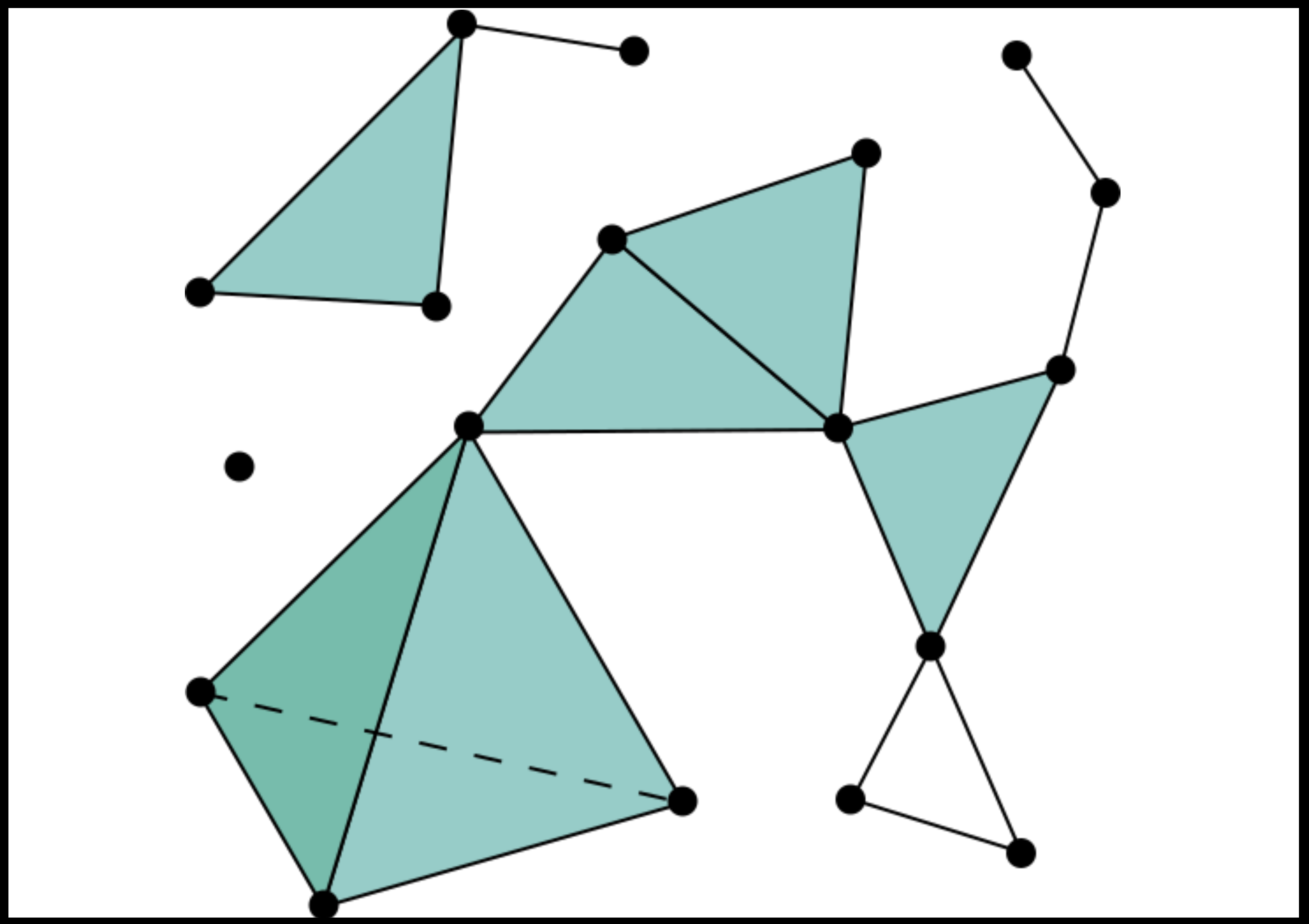
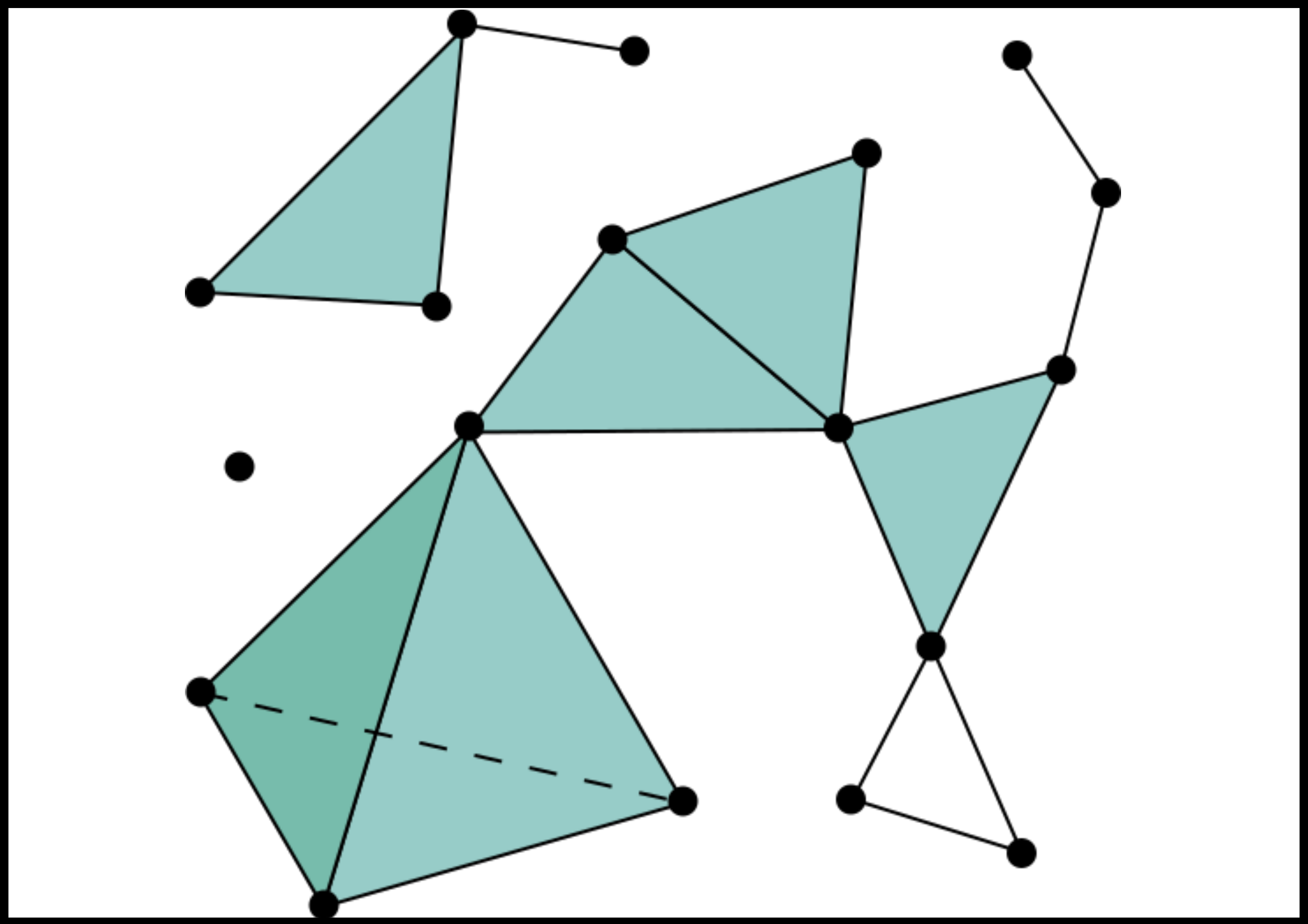


image credit: calm

Triangles, Tetrahedra and Topology



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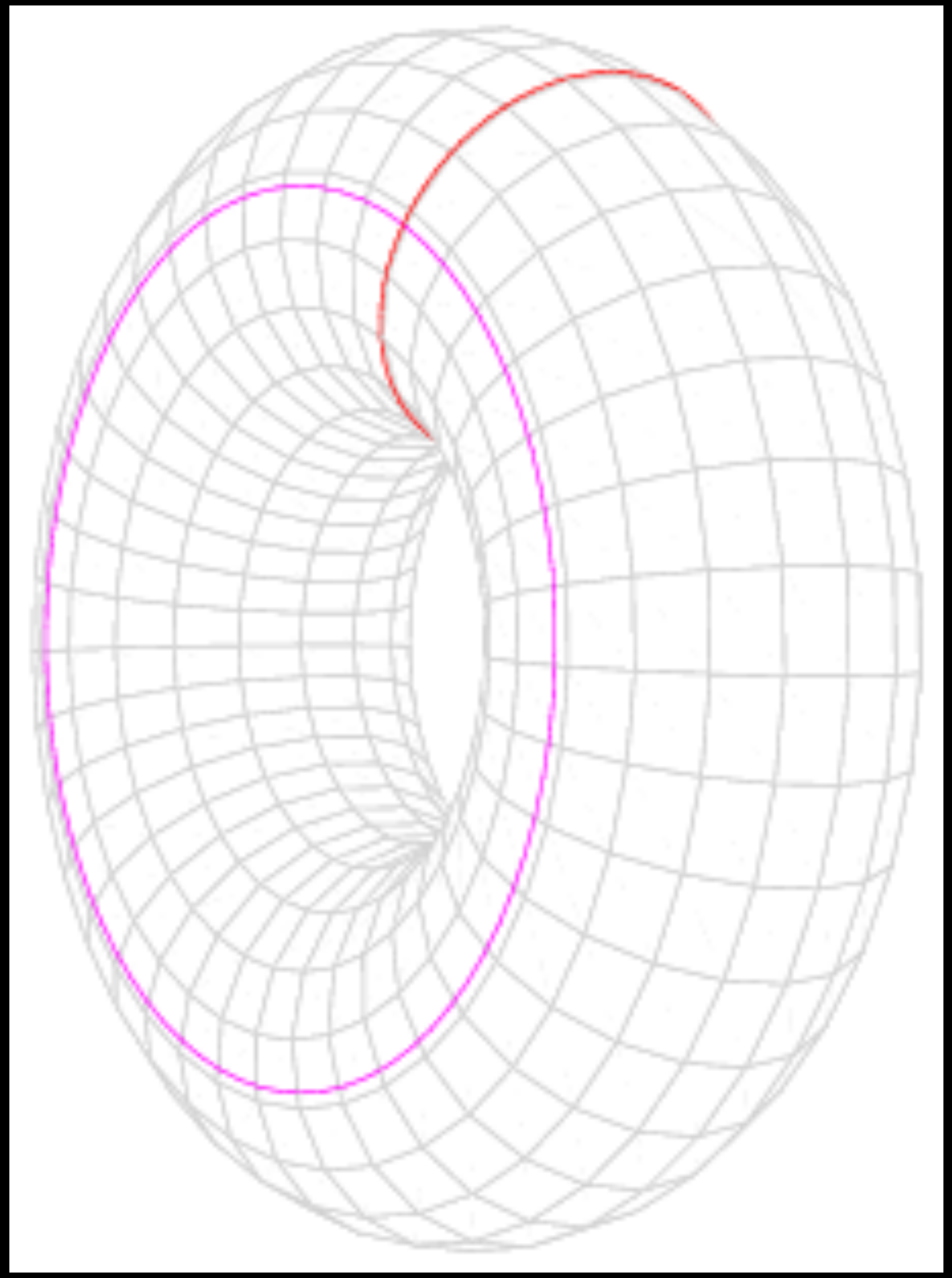

image credit: Krishnavedala

Who cares?

Examples of academic networks

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- Holes are repeated pathways. [Patania, Petri and Vaccarino 2017]

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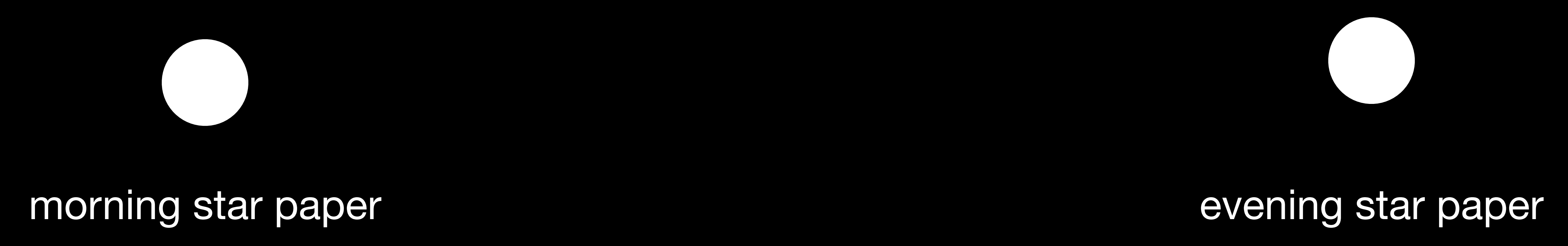
Examples of academic networks

- Holes are repeated pathways. [Patania, Petri and Vaccarino 2017]
- Unifying concepts fill holes. [Salnikov et al 2018]

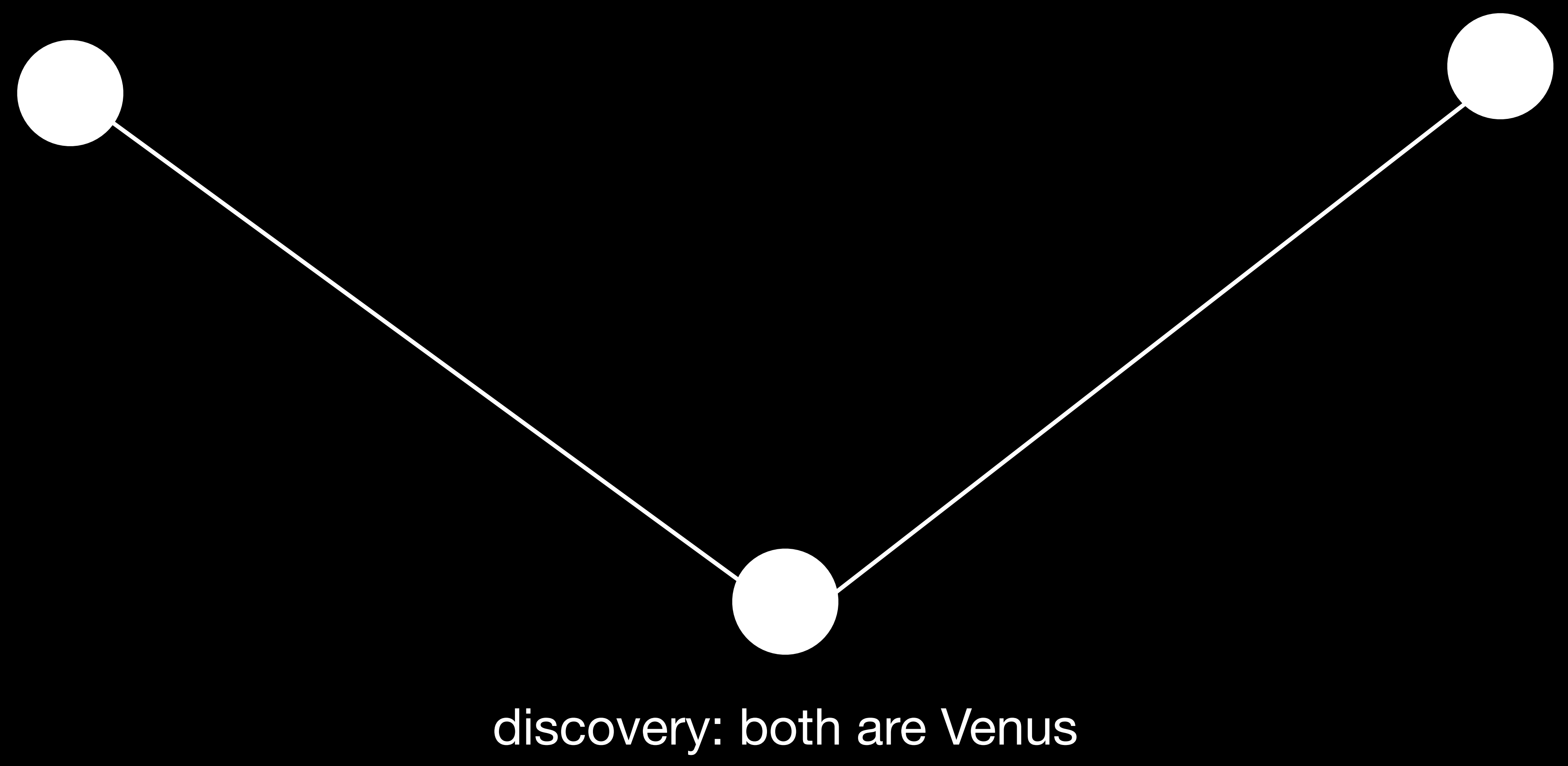
The Topology of the Citation Network

The story of Venus



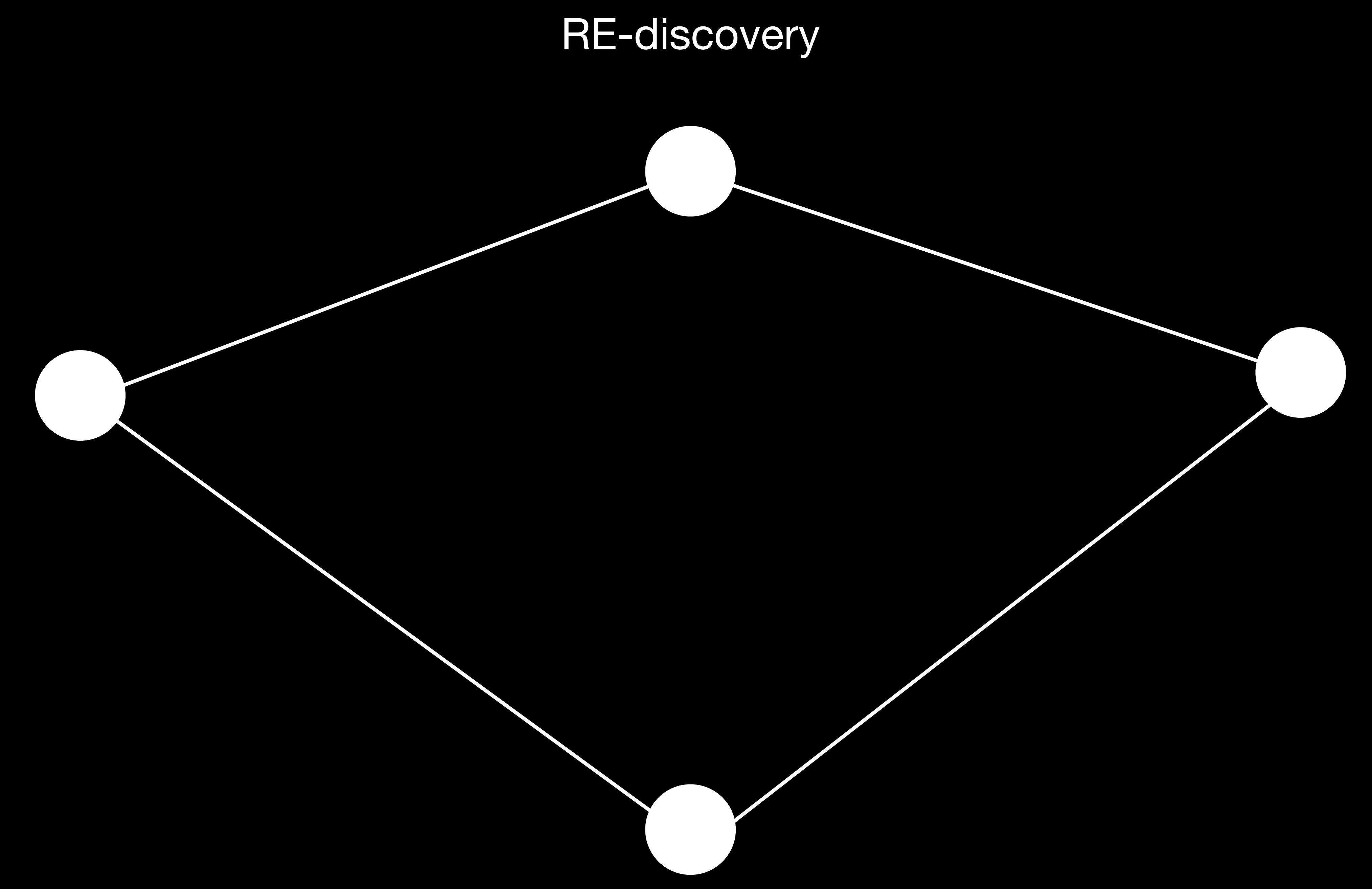
Unification merges components.

Venus’ citation network



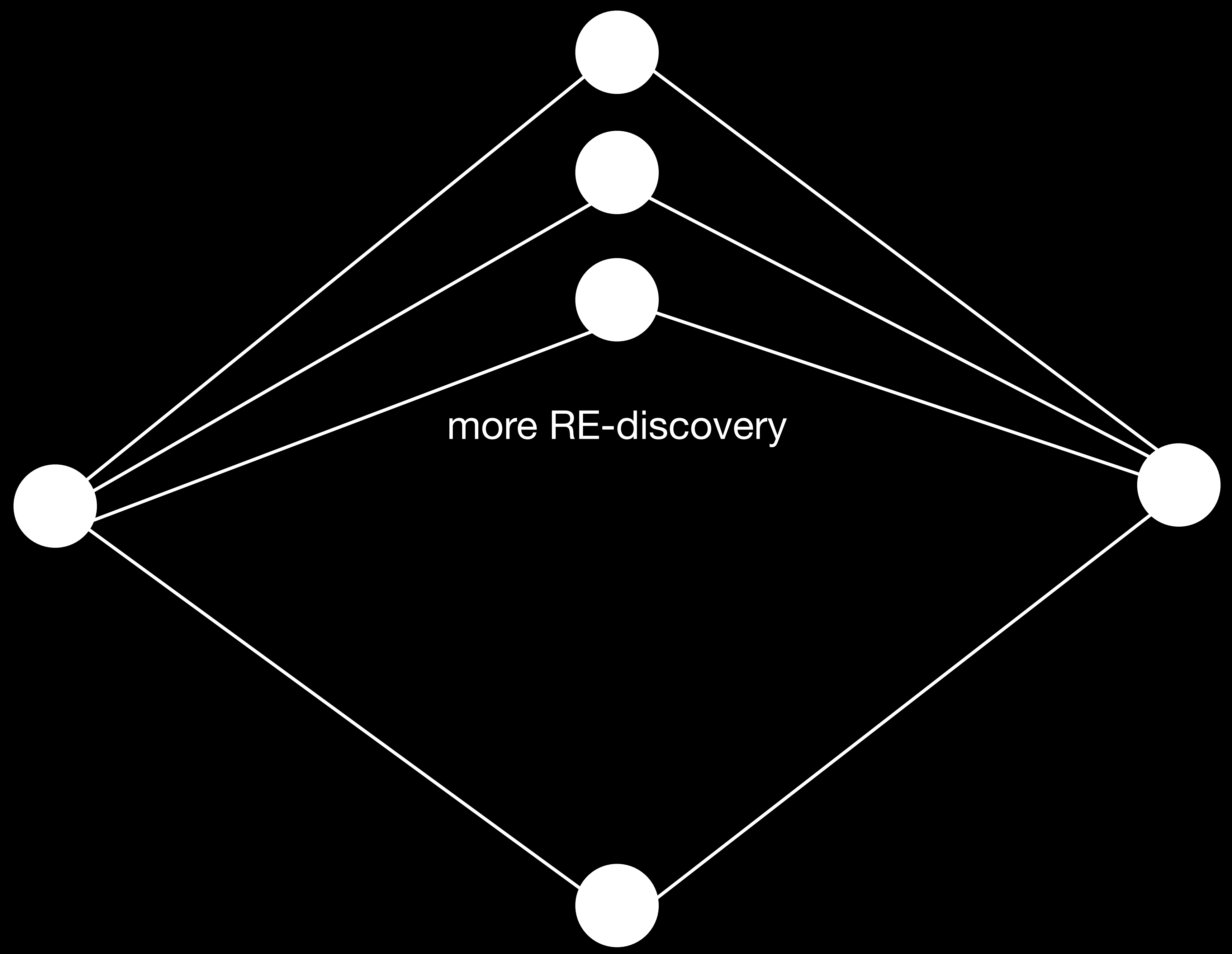
Re-unification creates a loop.

Venus' citation network



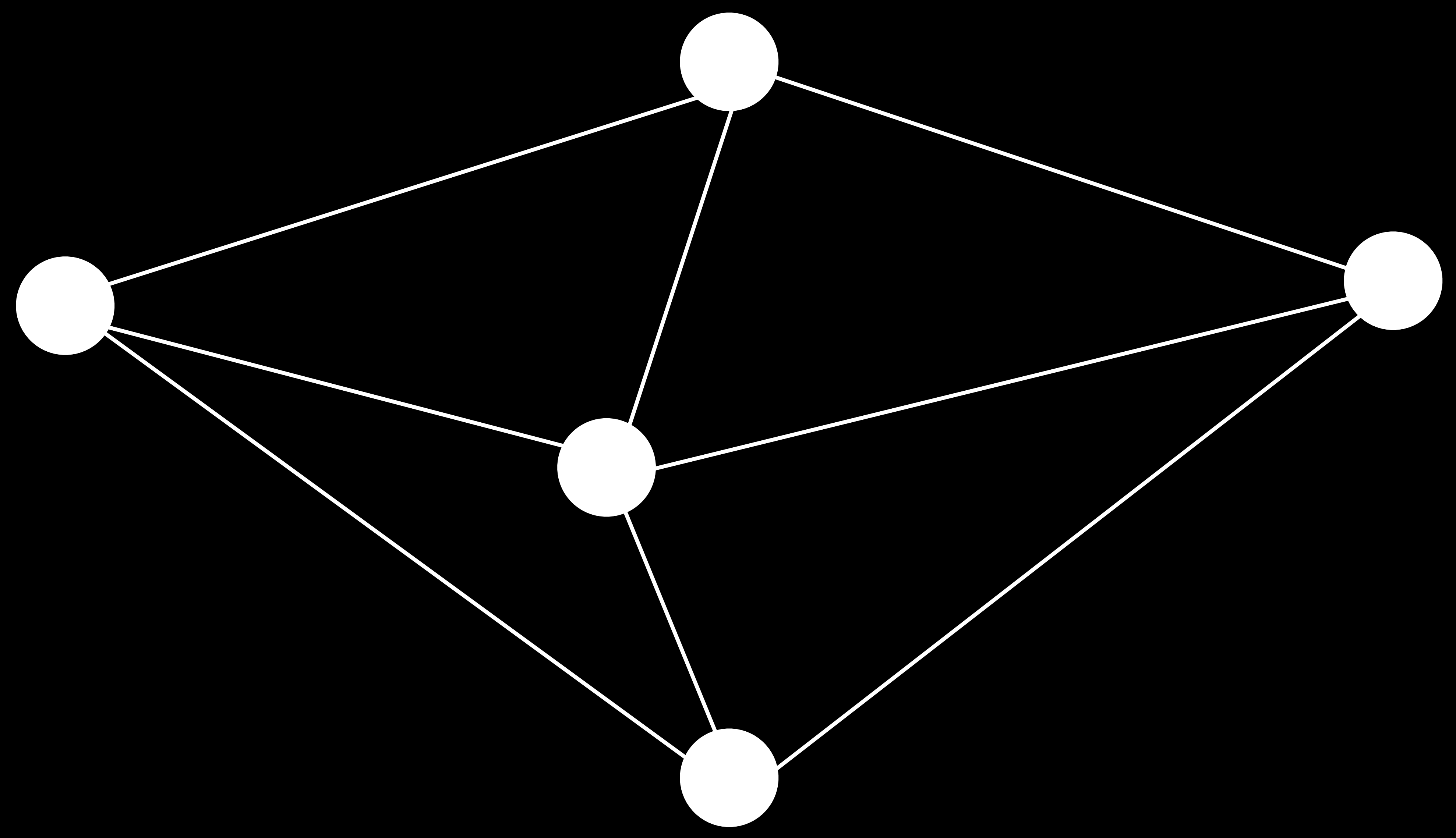
More re-unification creates more loops.

Venus’ citation network



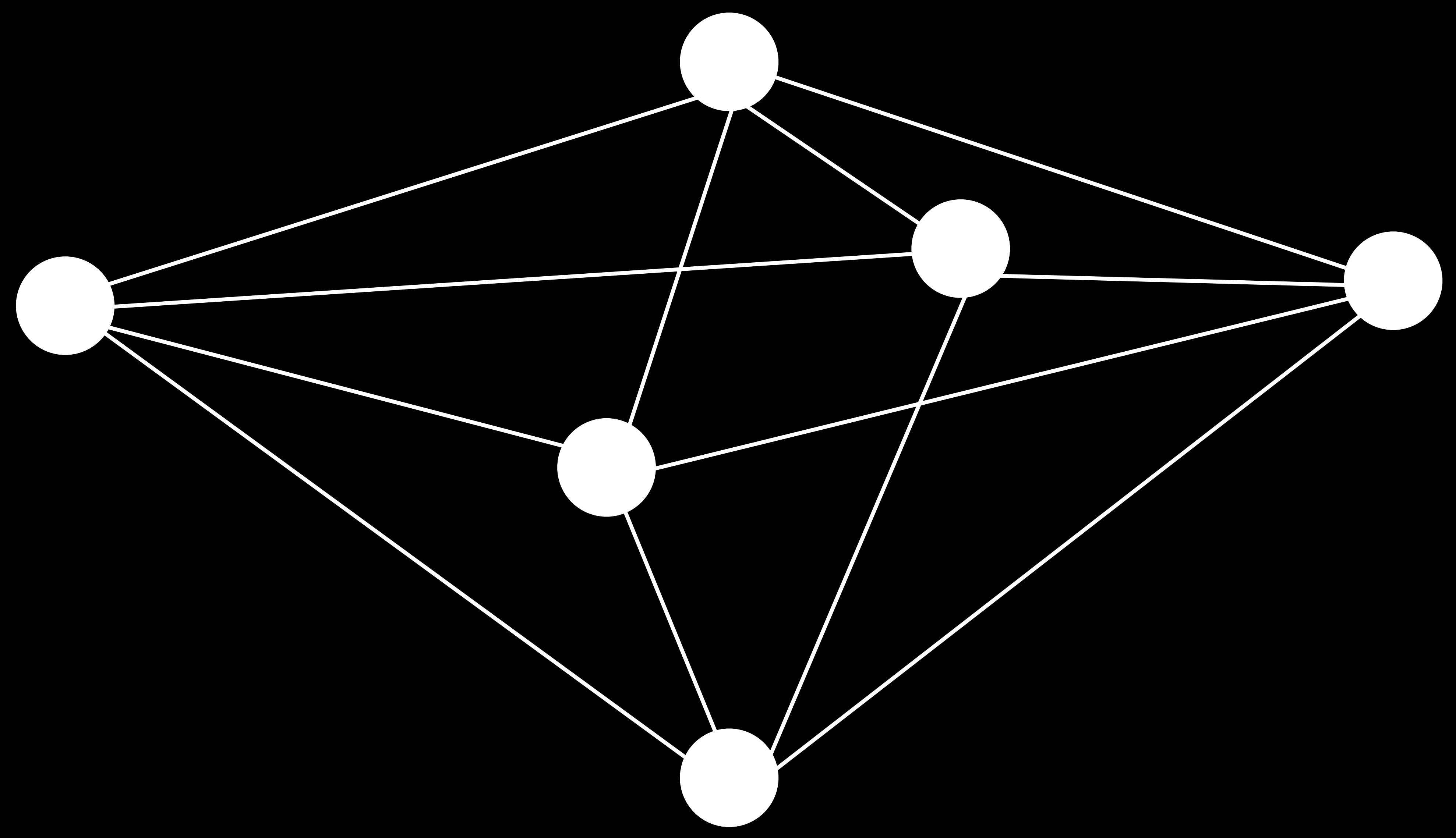
Hyper-unification reduces the number of loops.

Venus' citation network



Re-hyper-unification creates a cavity.

Venus’ citation network



betti numbers
= connected component and holes
= repeated higher-order connections

Betti numbers and preferential attachment

Preferential Attachment

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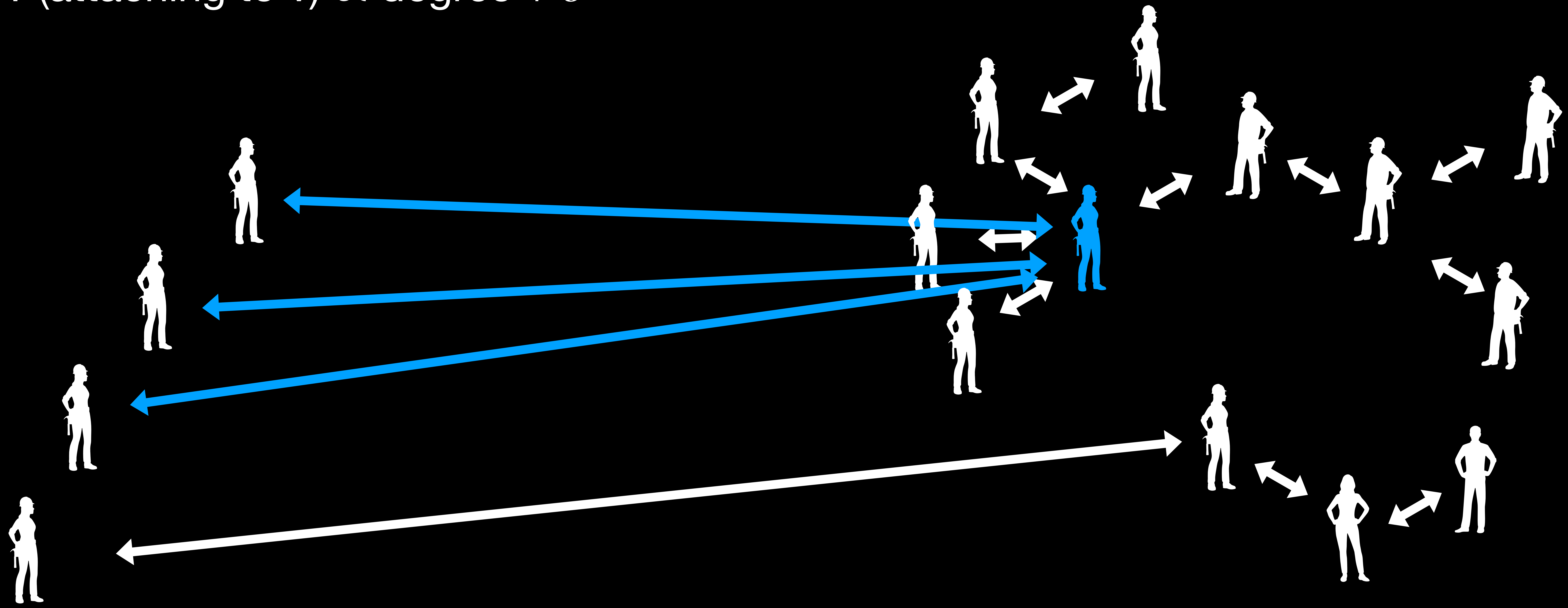
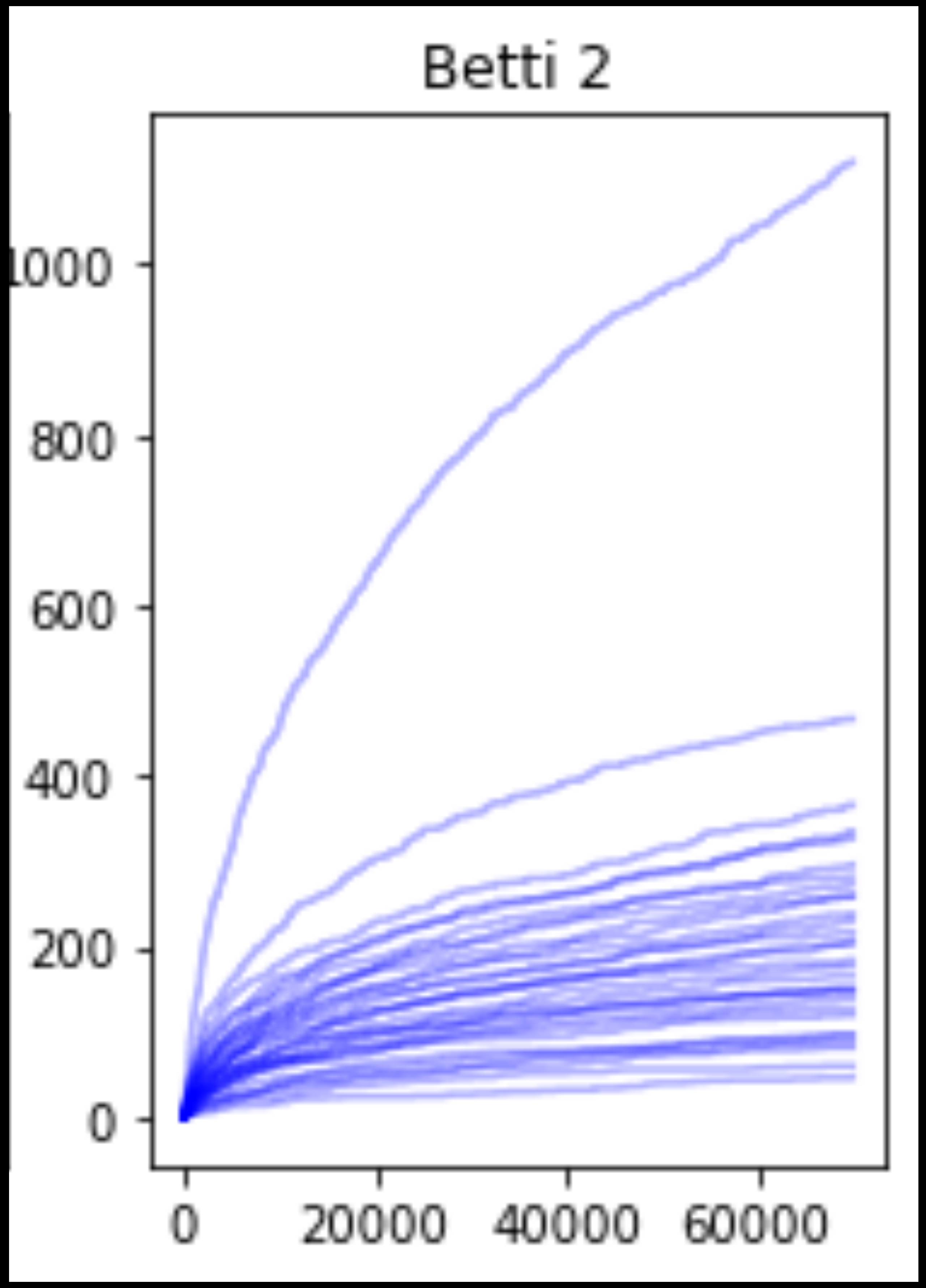
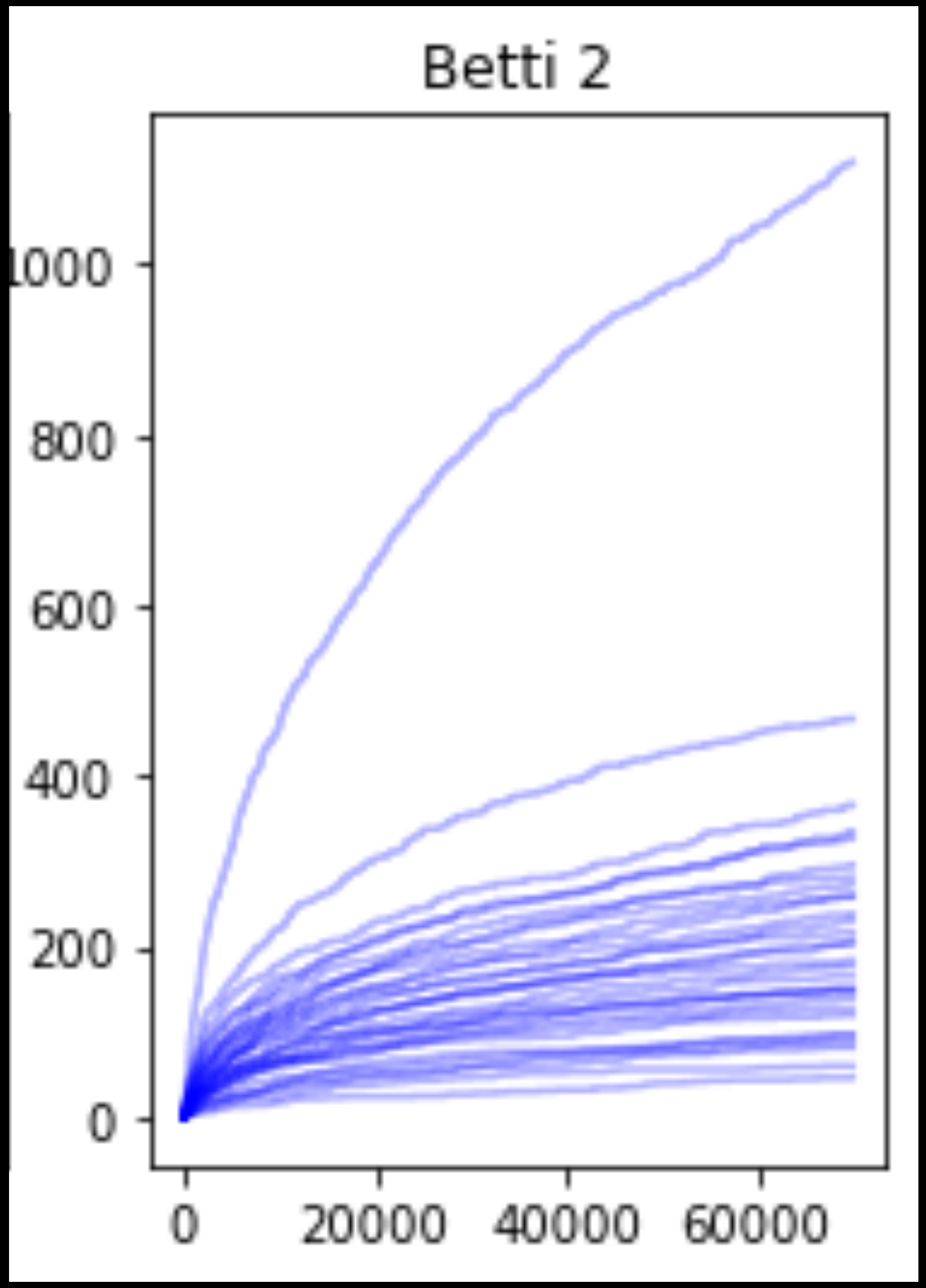


Expected Betti Number $E[\beta_q]$



Expected Betti Number $E[\beta_q]$

- $E[\beta_2] = \Theta(\text{num of nodes}^{1-4\chi})$ under mild assumptions
  - $\chi = 1 - \frac{1}{2 + \delta/m} \in (0,1/2)$
  - small χ:
    - heavier degree tail
    - stronger rich-get-richer effect



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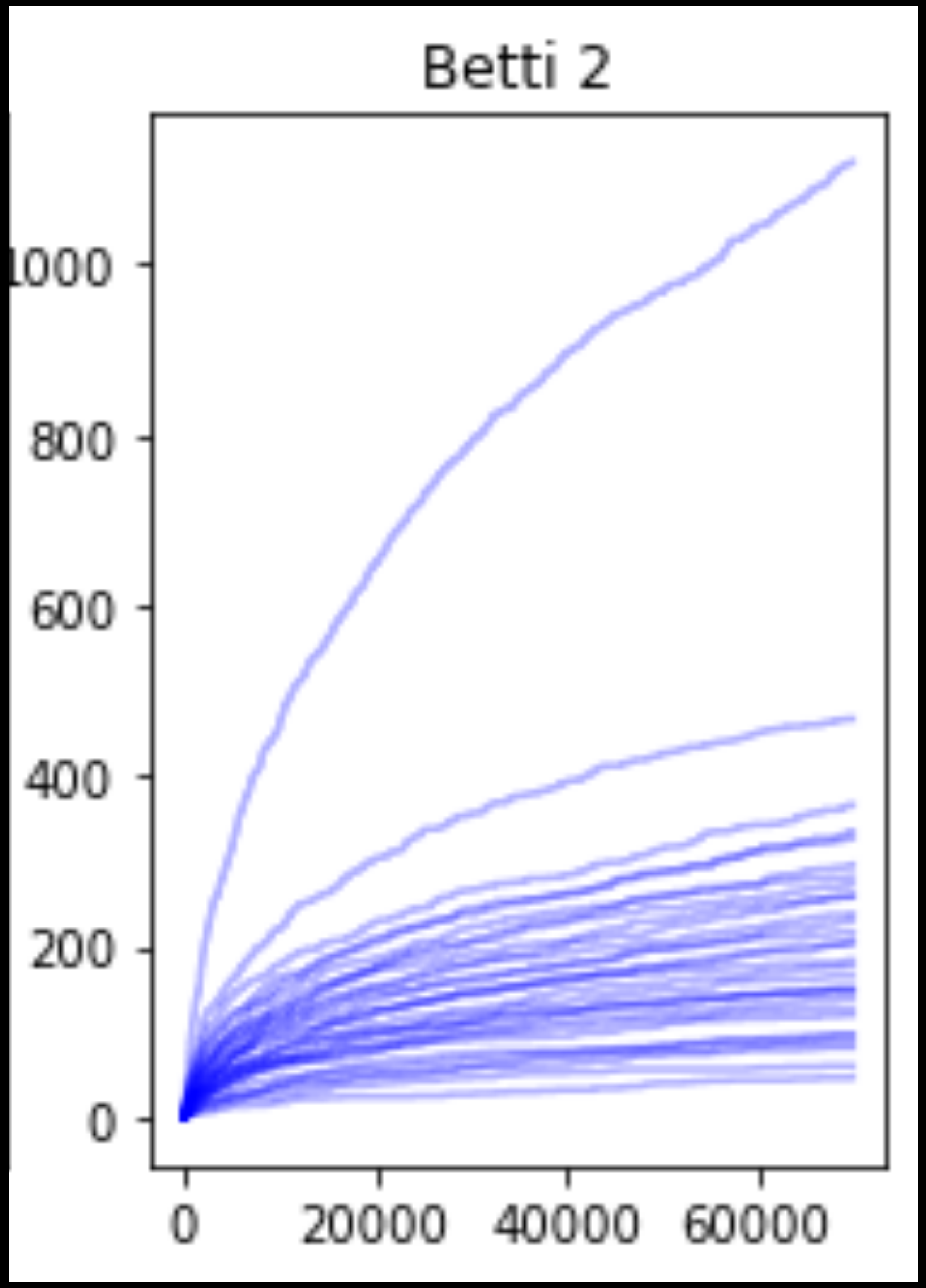

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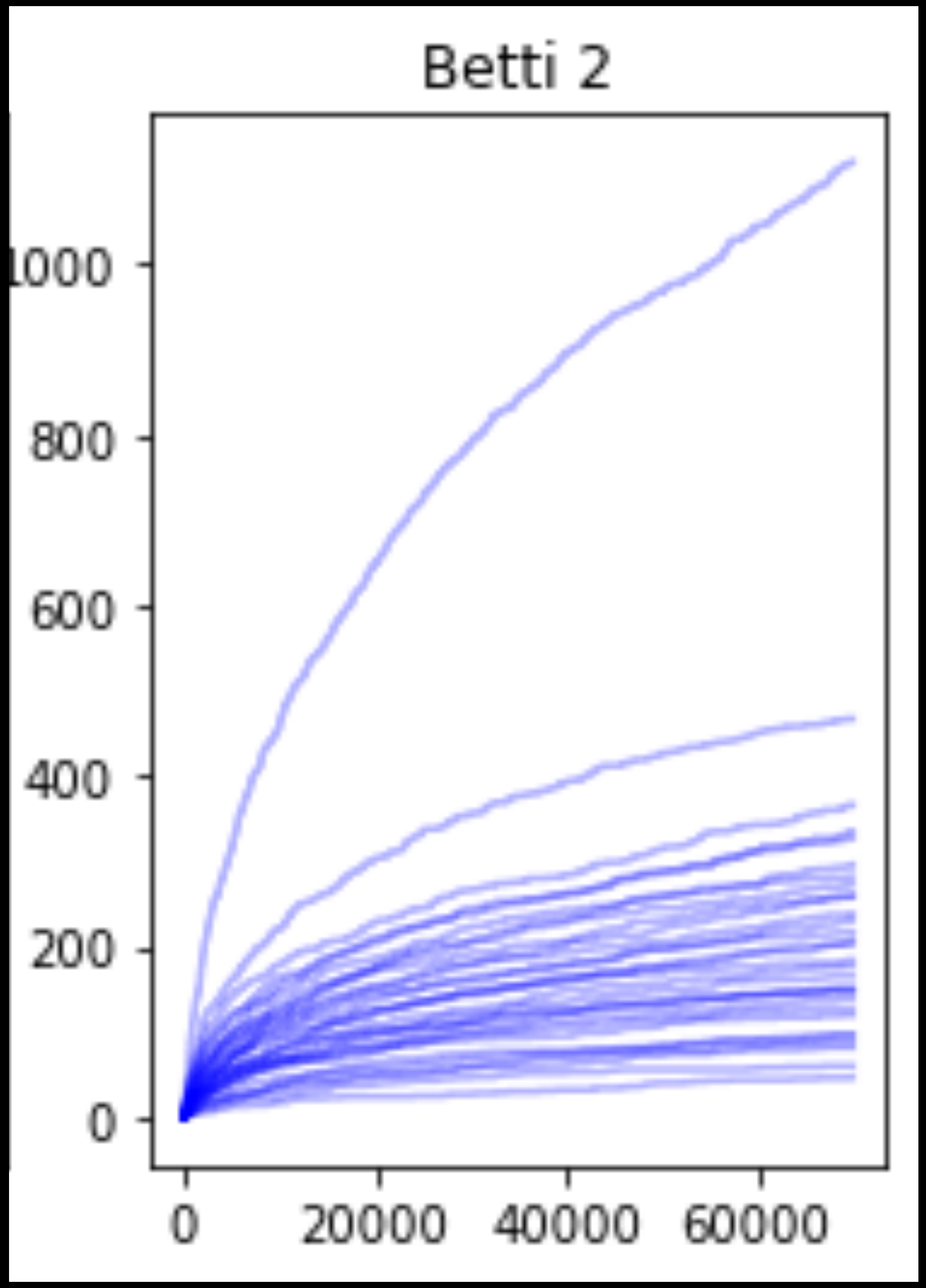
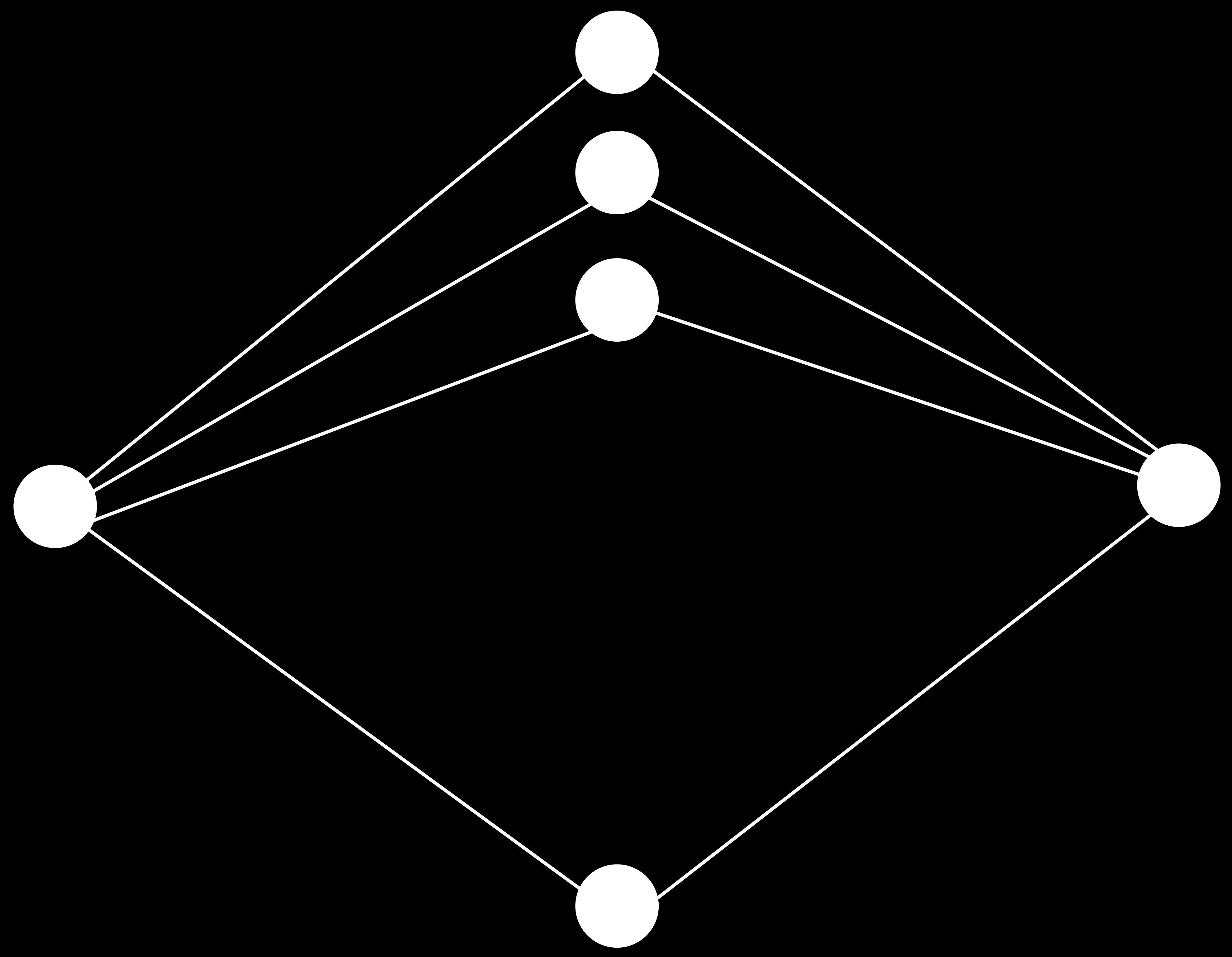
- $E[\beta_q] = \Theta(\text{num of nodes}^{1-2q\chi})$ for $q \geq 2$
- $E[\beta_q]$ decreases as dimension q increases
- $E[\beta_q]$ increases with the rich-get-richer effect



Main Idea



What’s next?

- Tail of betti numbers?
- robustness and betti numbers?

Thank you

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