Betti Numbers of Preferential Attachment Complexes

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Preferential Attachment Clique Complexes

•Inductively built **random graph** with ***T* nodes**
•Each node *v* connected to ***m* previous nodes**
•$P(v \to j) \propto \deg j + \delta$, with **tuning parameter** $\delta \in (-m, 0)$
•Collapse repeated edges and build **clique complex**

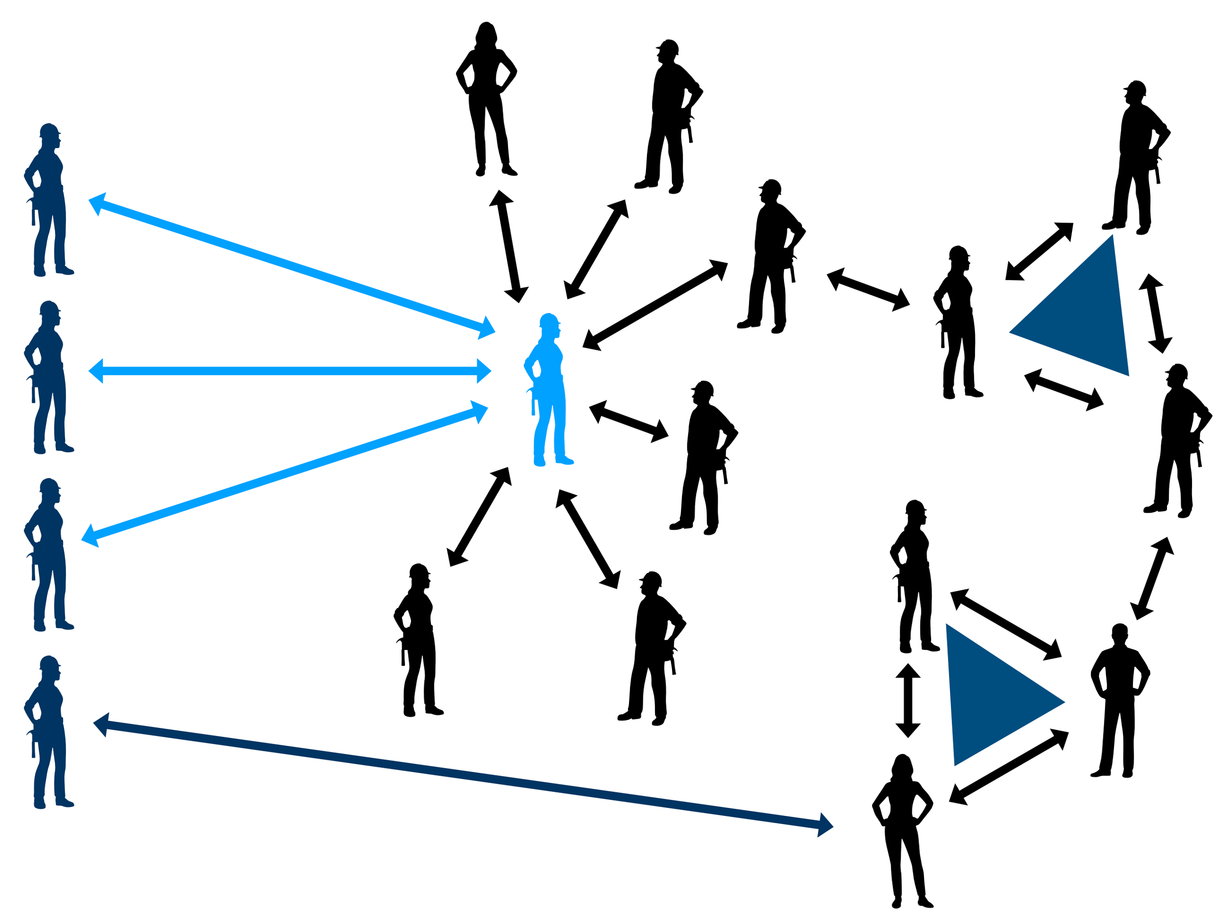
Evolution of Betti Numbers

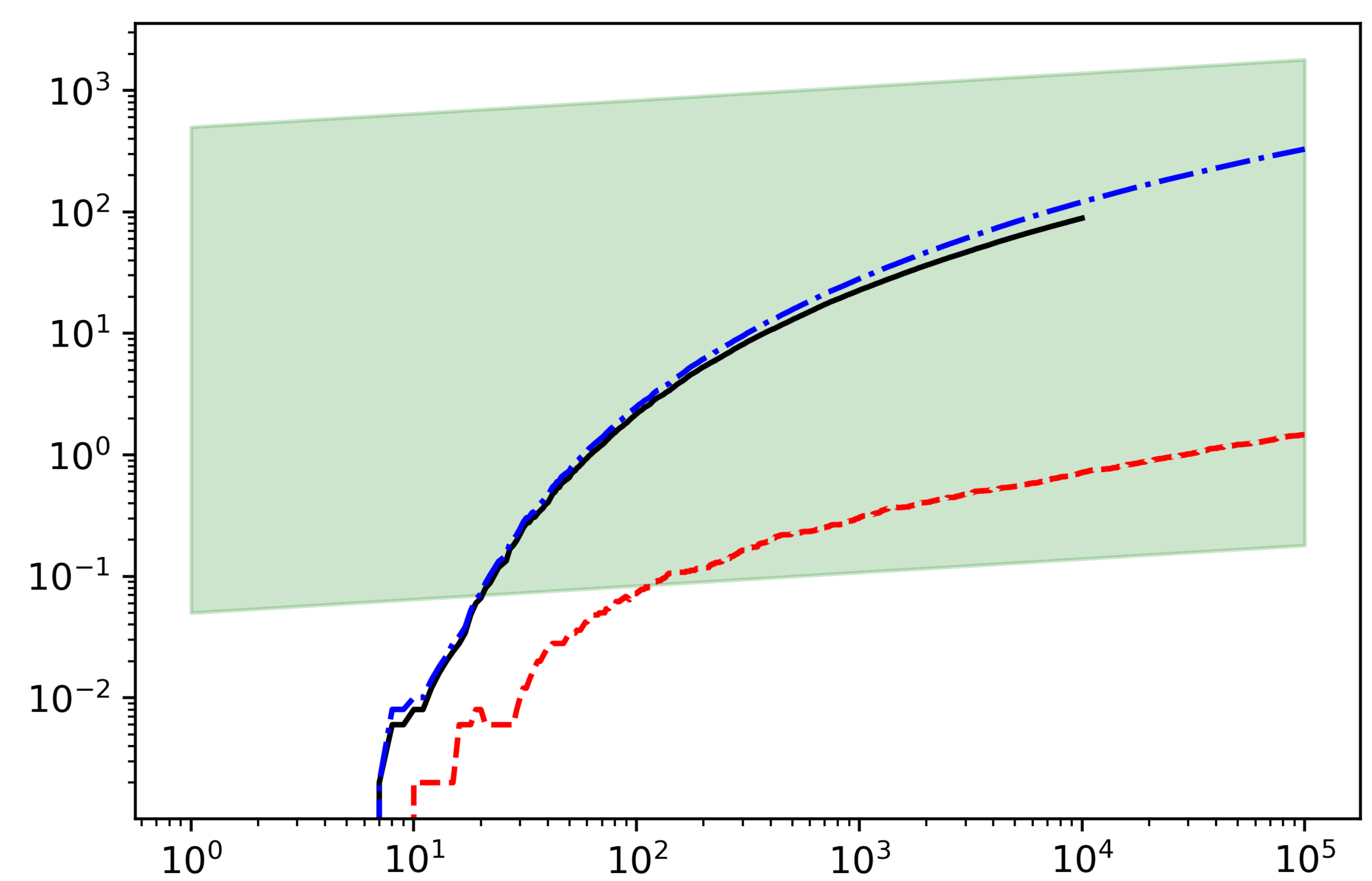
Dimension 1 $E[\beta_1] = (m-1)T + o(T)$

Higher Dimensions Let $\chi = 1 - (2 + \delta/m)^{-1}$. For $q \geq 2$,

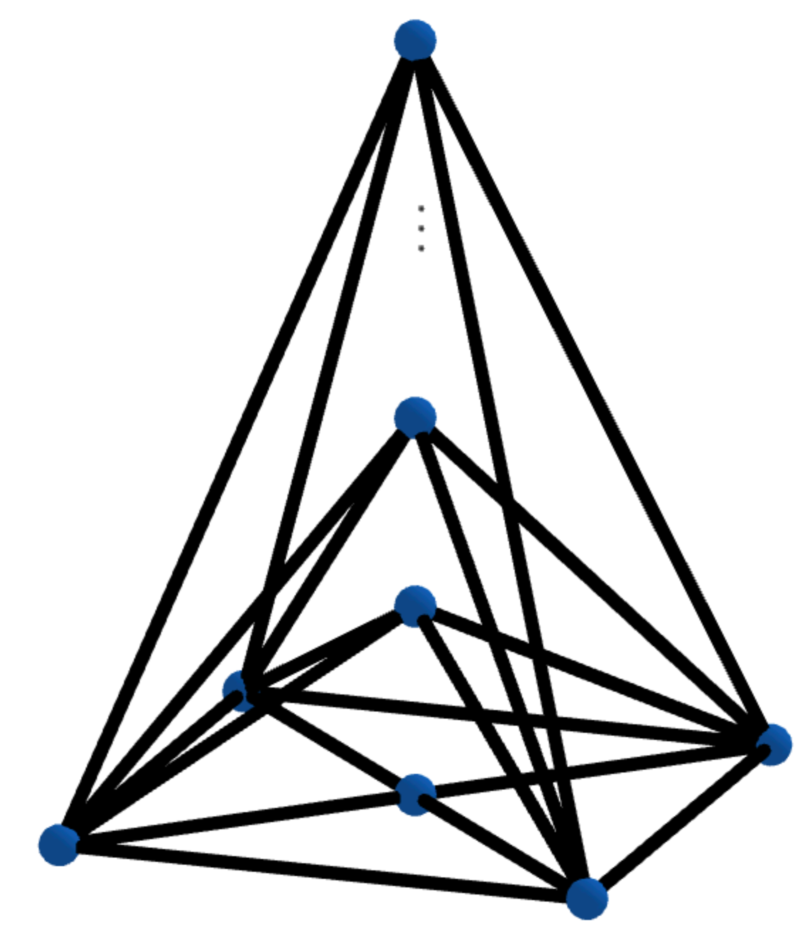
$$cT^{1-2q\chi} \leq E[\beta_q] \leq CT^{1-2q\chi}$$

for some constants *c*, *C* > 0 if $1 - 2q\chi > 0$ and $m \geq 2q$.





The evolution of the average β_2 as the number of nodes increases
The dotted curves are bounds on the average β_2
The slope of the shaded region is the asymptotic slope



repeatedly coned squares

Intuition?

•**Coned squares** dominate. So do their higher-dimensional analogues.
•Boundaries are **rare** because they are more **complicated**.

In Human Language?

•**Sublinear** growth
•Gradually decreasing **topological complexity**
•Complexity increases with the **rich-get-richer** effect.

How to Show?

•Localize the computation with a **mapping cone** argument.
•Characterize cycles by a **minimal-cycle** [Kahle 2009] argument.
•Apply **graph-counting** [Garavaglia and Steghuis 2019] arguments.

What's Next?

•Tail behavior?
•Computable local invariants?

Cited Works on this Poster
•Garavaglia A. and Steghuis C.: Subgraphs in preferential attachment models. *Advances in Applied Probability*,51(3), 898 — 926 (2019).
•Kahle M.: Topology of random clique complexes. *Discrete Mathematics*, 309(6): 1658 — 1671.